Reconstruction of Evolving, Non-Convex Curves from a Sequence of Single-Angle Projections

-- or --

Radiographic Reconstructions Utilizing the Dynamics Connecting Differing Times

---

Thomas J. Asaki and Kevin R. Vixie

Los Alamos National Laboratory, Los Alamos, NM

Erik M. Bollt

Clarkson University, Potsdam, NY

Simulation and Analysis Workshop -- Part 1: Tools

August 5, 2002

The Exploratory Concepts

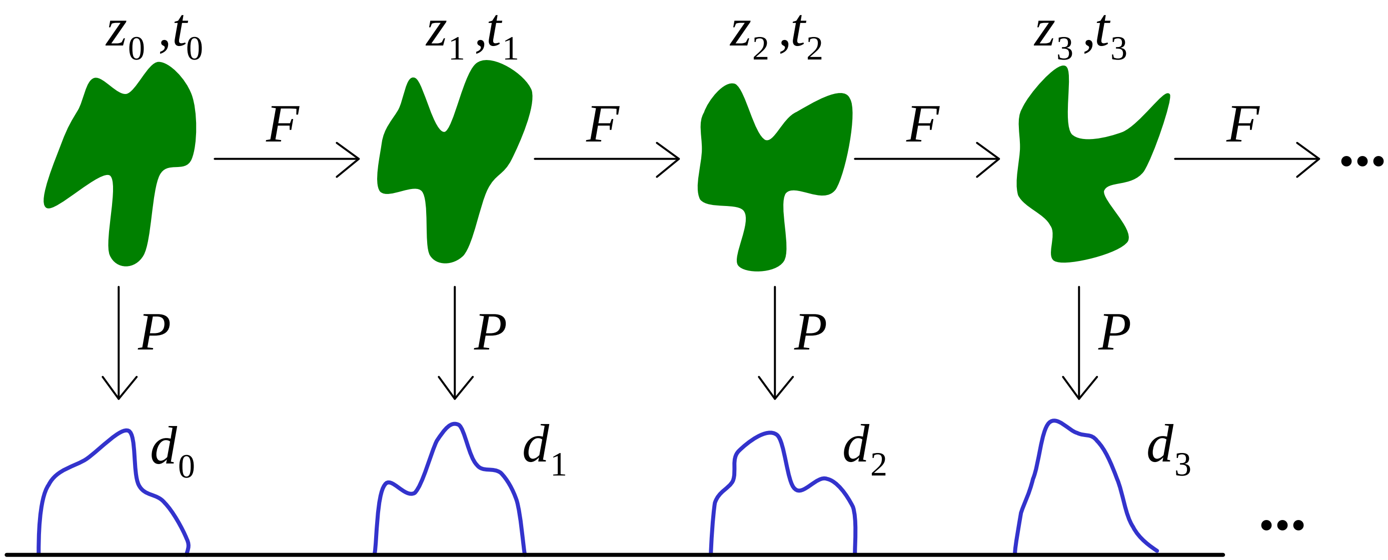
Can a time-sequence of single-view (or few) radiographs be used to enhance object reconstructions through dynamics constraints?

How much knowledge is required of the dynamics that connect different times?

Can the nature of the dynamics be determined simultaneously along with the object description?

How are these results affected by computational time, error propagation, and metrics?

Incorporating Dynamics in Reconstructions



z_0 is some initial object

$z_k = F^k z_0$ is the evolved object at time t_k

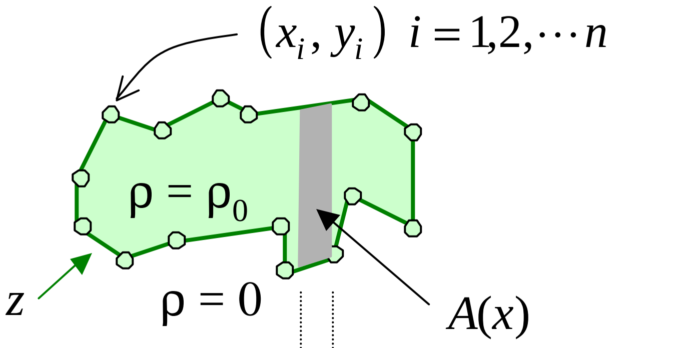
$d_k = P\, z_k$ is a low-dimensional projection of object z_k

$F = F(\lambda, t)$ is a time evolution operator

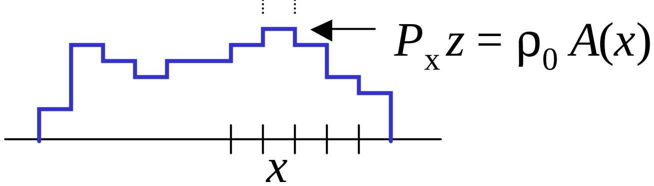
f = merit function $\quad f = \sum_k \left\| d_k - PF^k z_0 \right\|$

A First Look

The Object:
A uniform-density simply-connected object defined by a set of ordered points



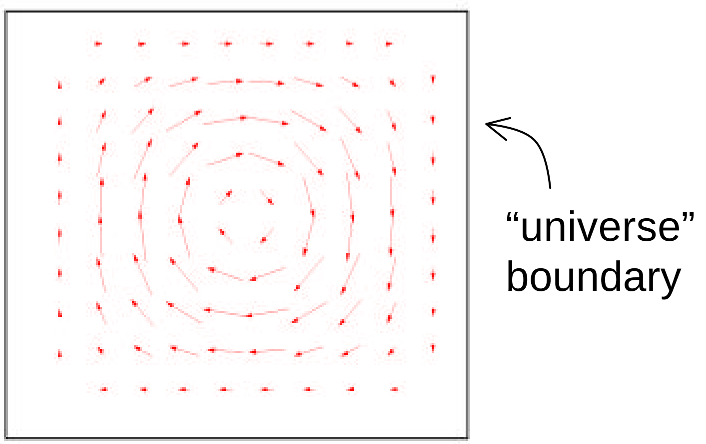
The Projection Data:
Single-view discrete mass projection, possibly with simulated noise



The Dynamics:
A vortex-like fixed advection flow

$$\dot{x} = -\sin^2(x)\sin(2y)$$

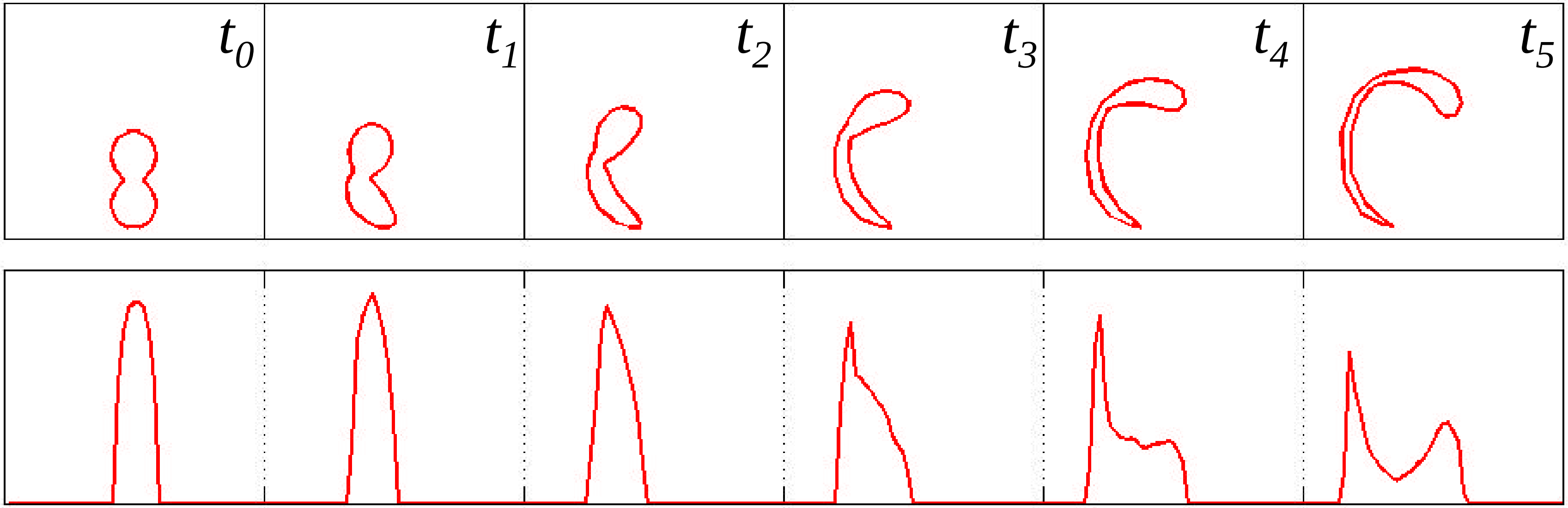
$$\dot{y} = +\sin^2(y)\sin(2x)$$



Example Experiment and Data

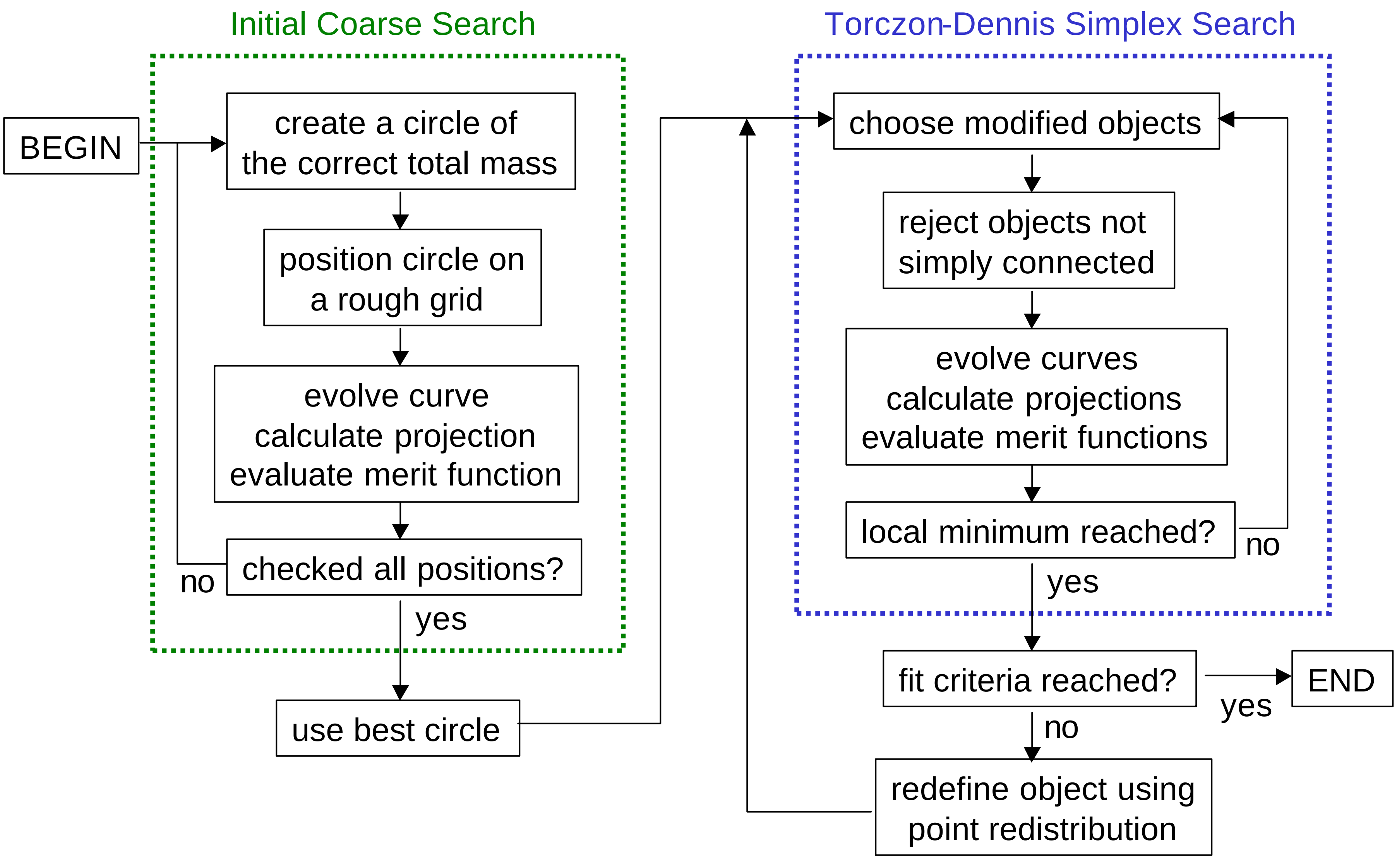
The object is defined by 30 vertices.
Each of six time views has 50 sampling bins.
The dynamics is exactly known.

Evolving object at six different times

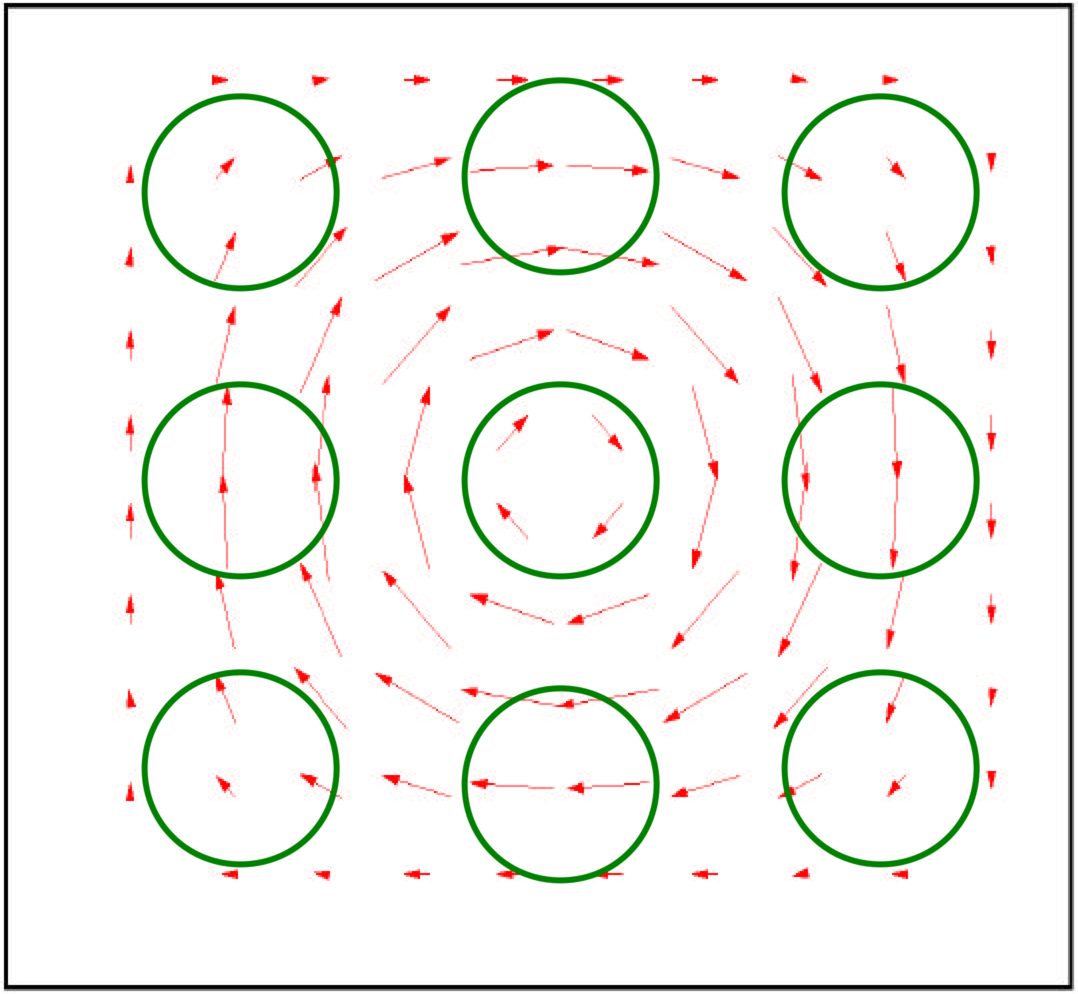


Corresponding projection data (noiseless case)

The Algorithm



Initial Coarse Search

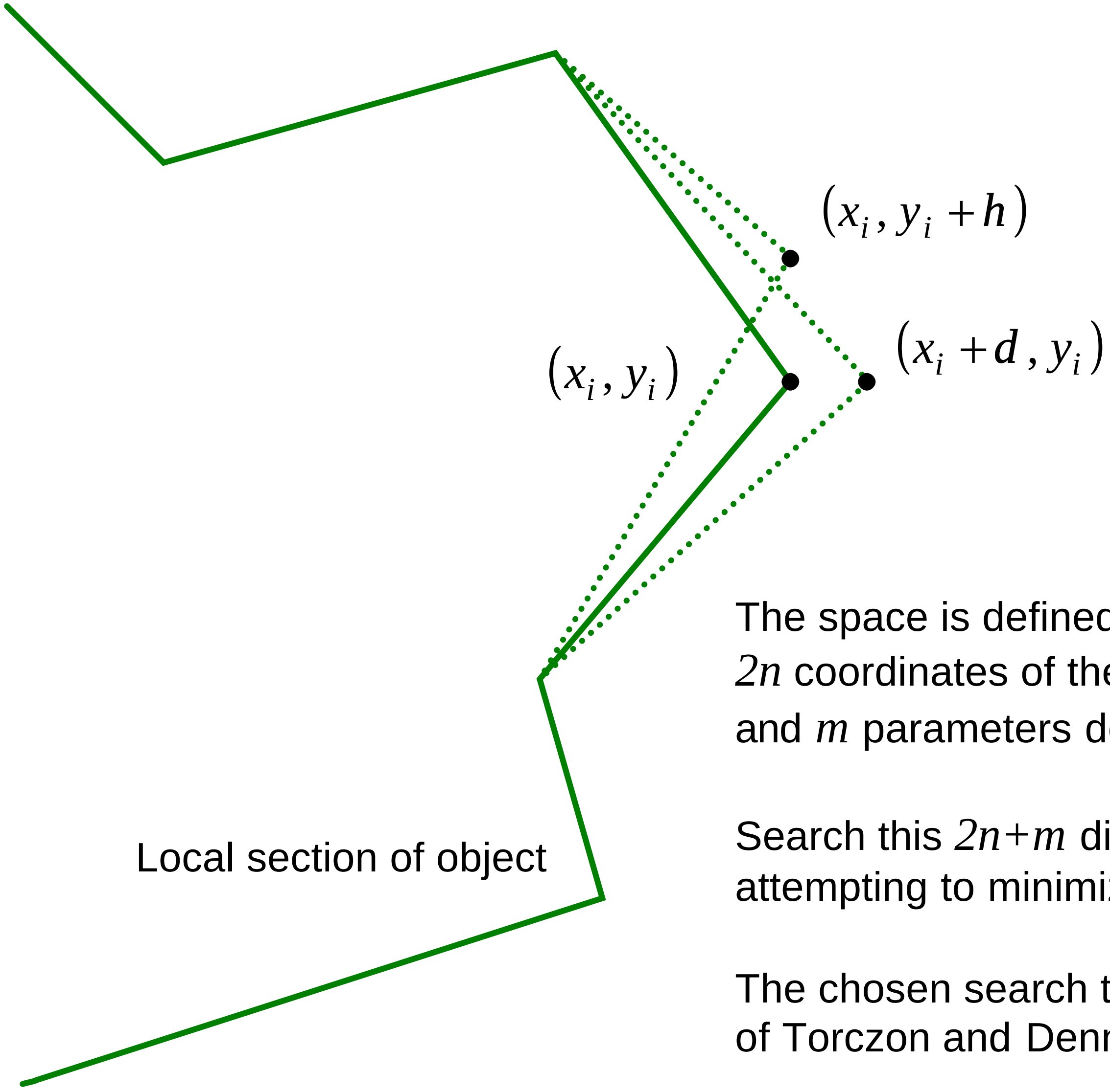


Begin with an initial circular object of the correct mass (regular polygon of n sides).

Search over small set of initial locations in the universe -- computing projections and evaluating the merit functions.

This quick and simple process determines the most likely initial location for the unknown object.

The Refined Search

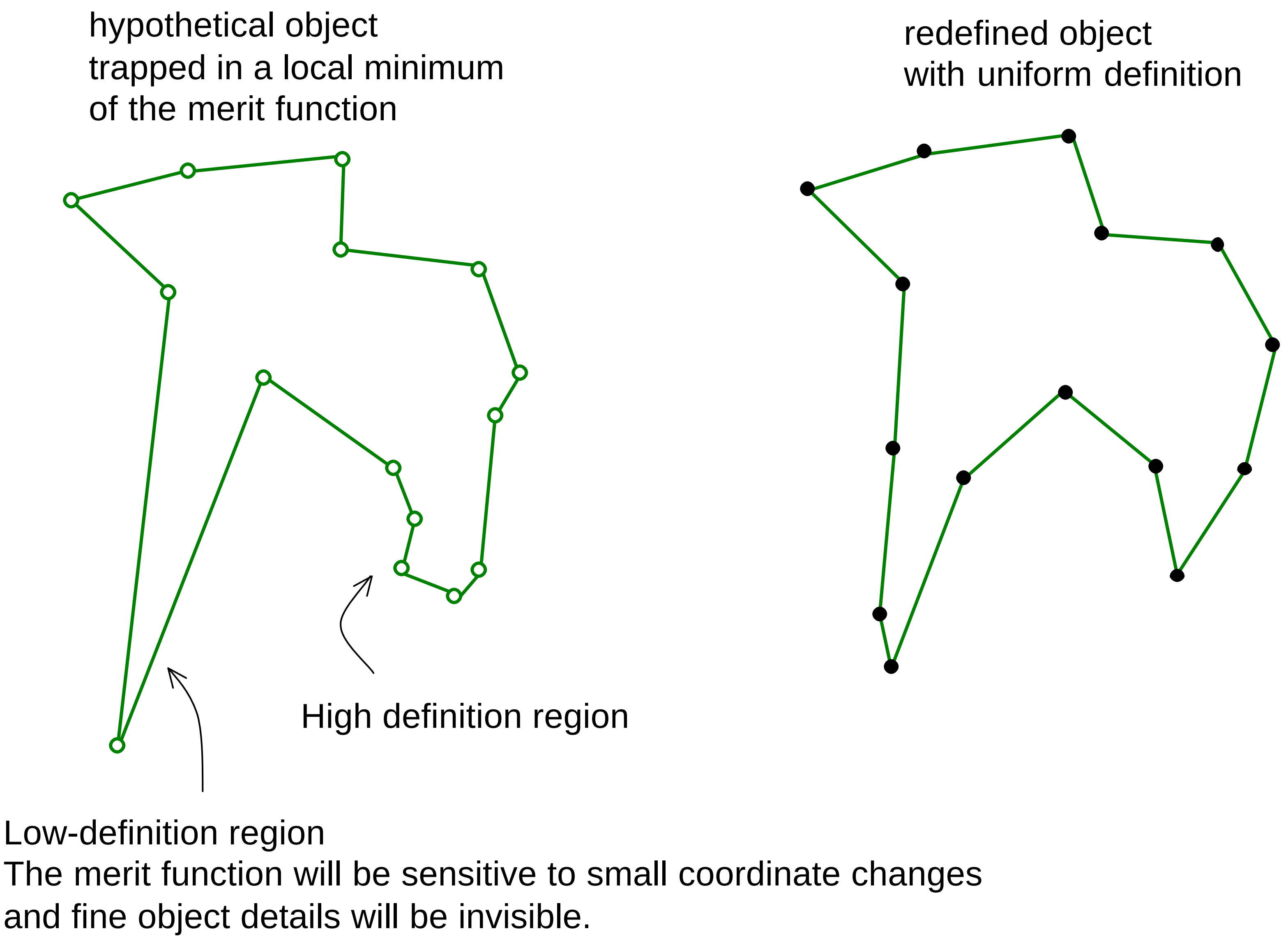


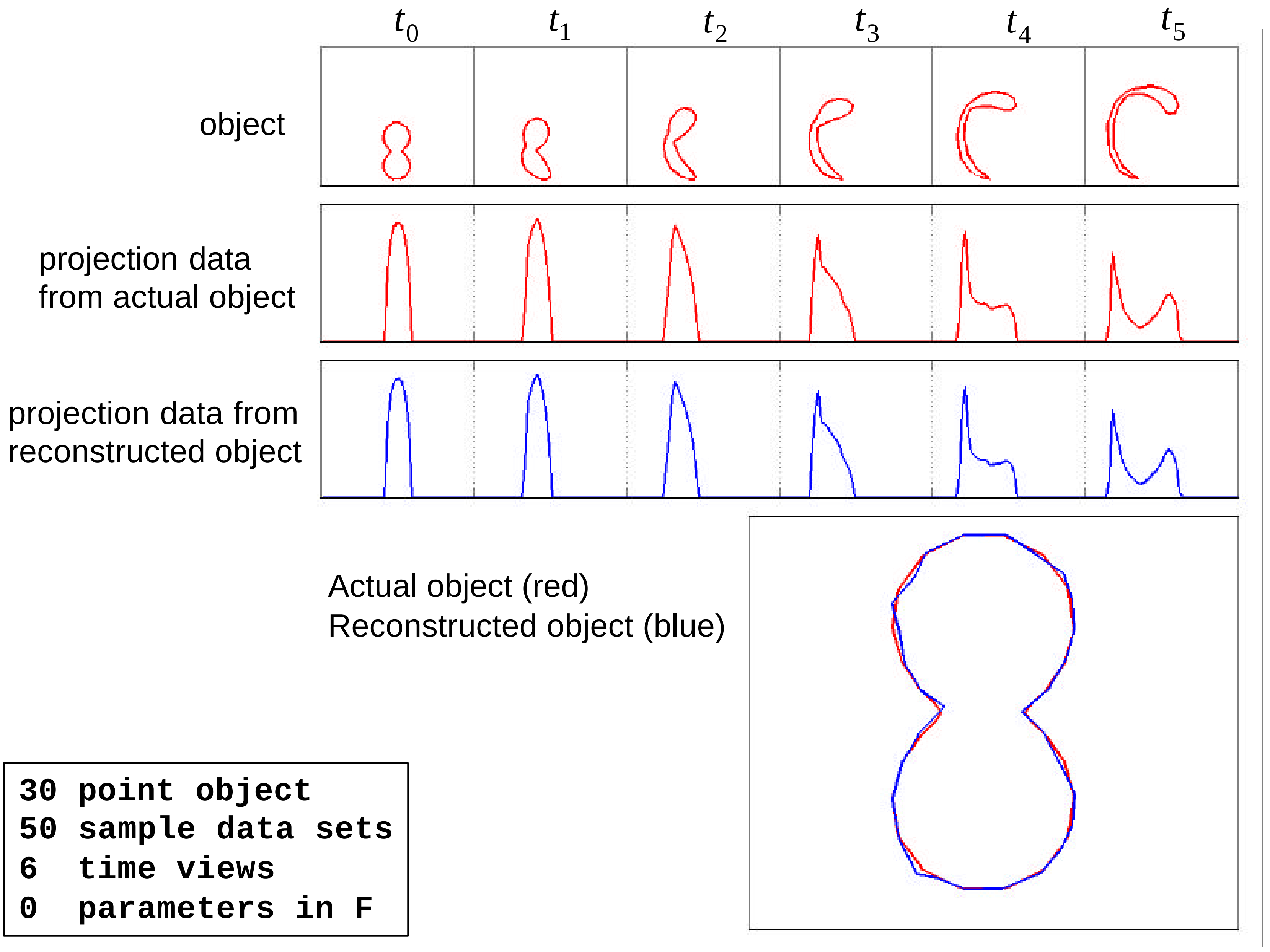
The space is defined by:
$2n$ coordinates of the vertices of the initial object
and m parameters defining the evolution dynamics.

Search this $2n+m$ dimensional space
attempting to minimize over the merit function.

The chosen search technique is the simplex method
of Torczon and Dennis.

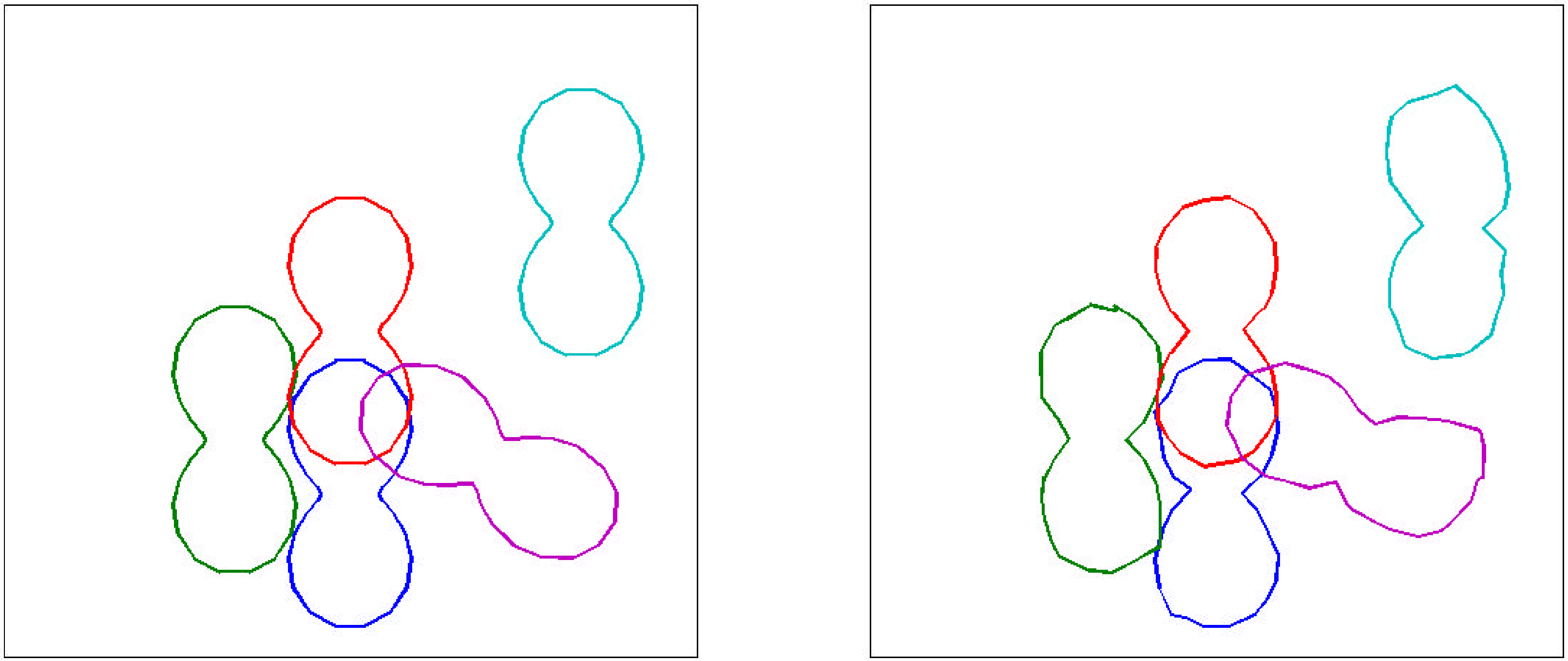
Example of Object Redefinition
(exiting local minima of the merit function)





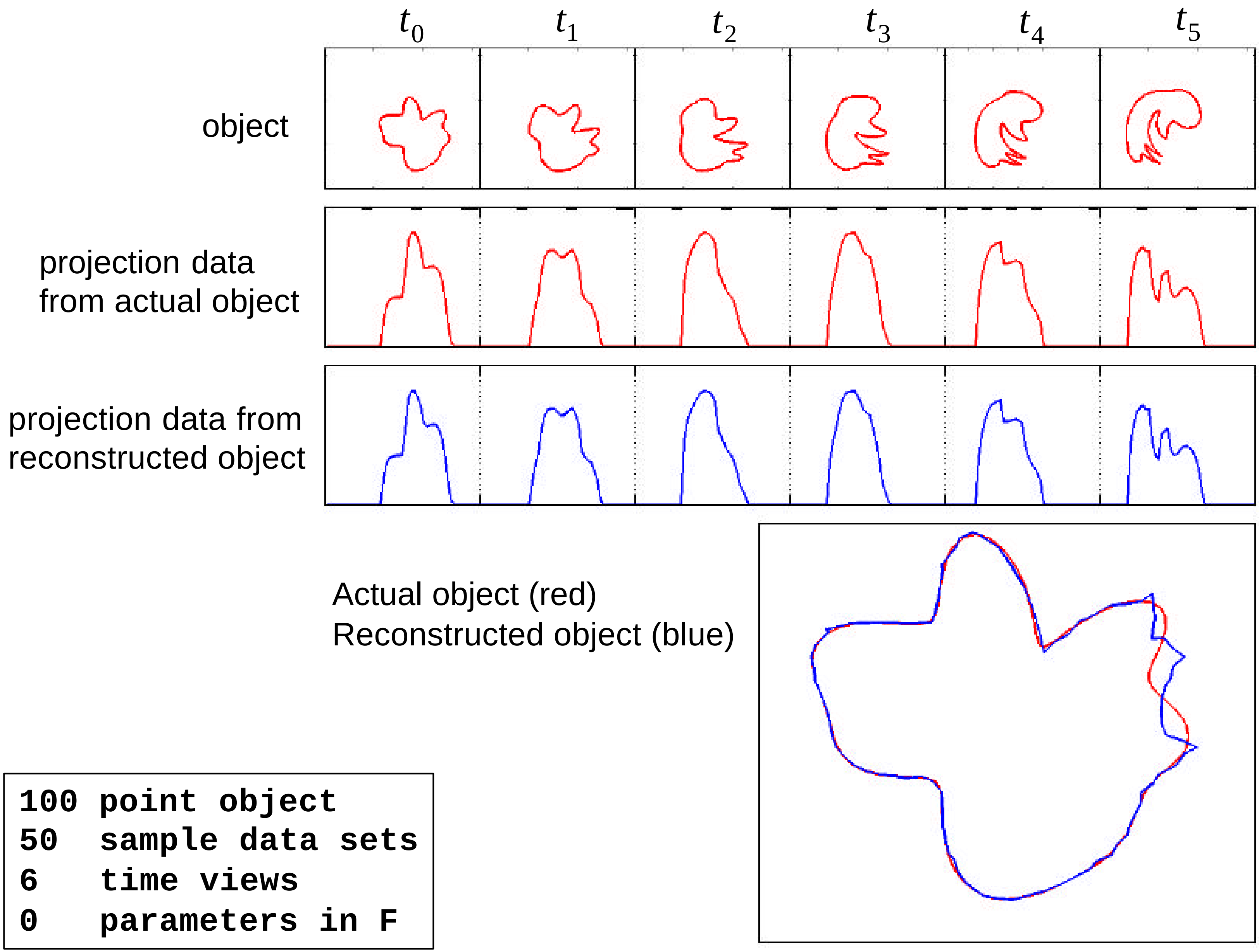
30 point object
50 sample data sets
6 time views
0 parameters in F

Identical Objects With Varying Initial Position and Orientation

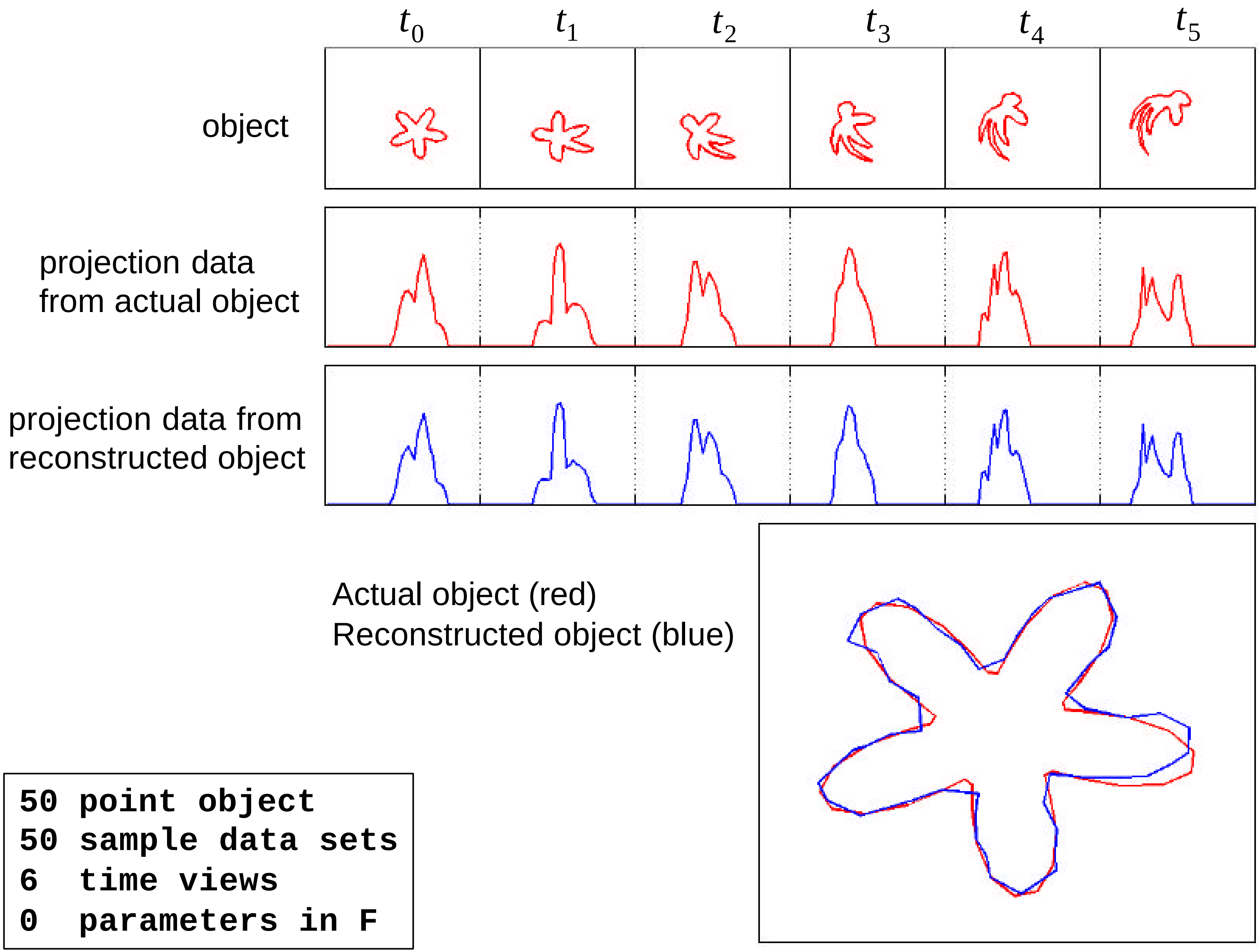


Objects

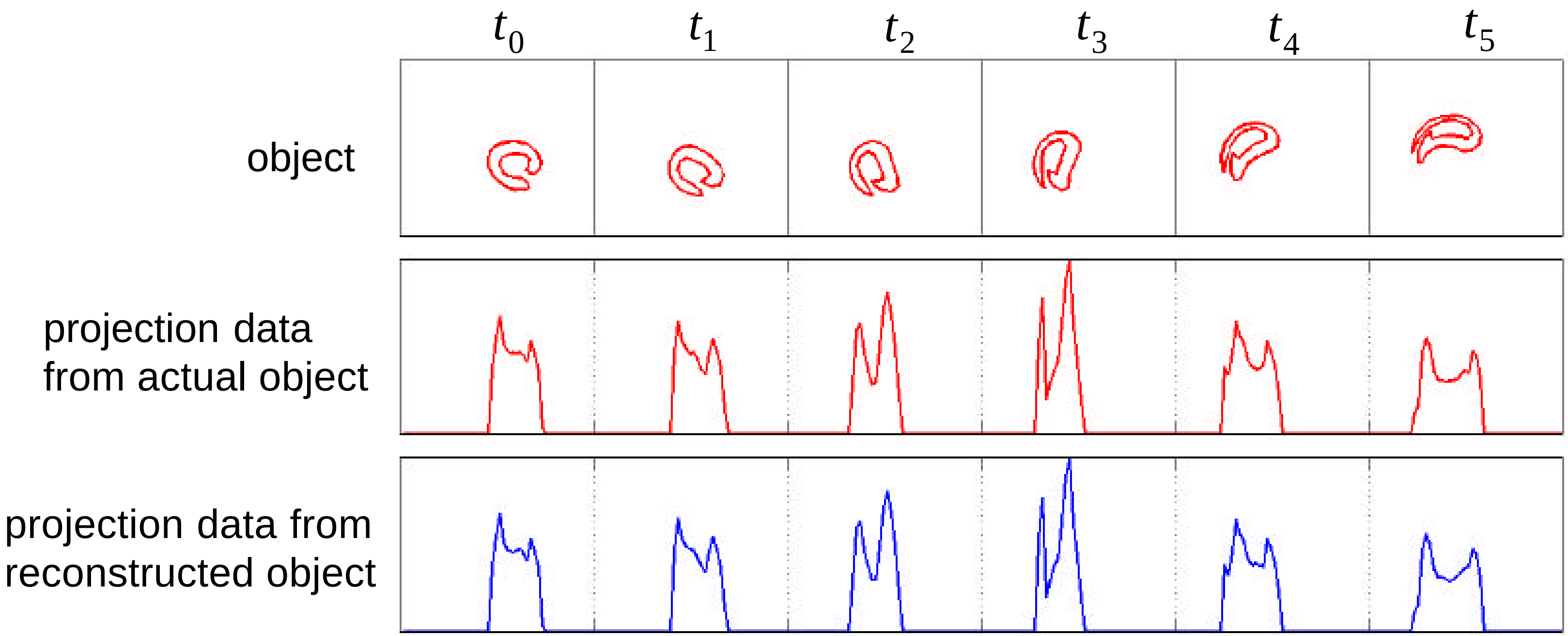
Reconstructions



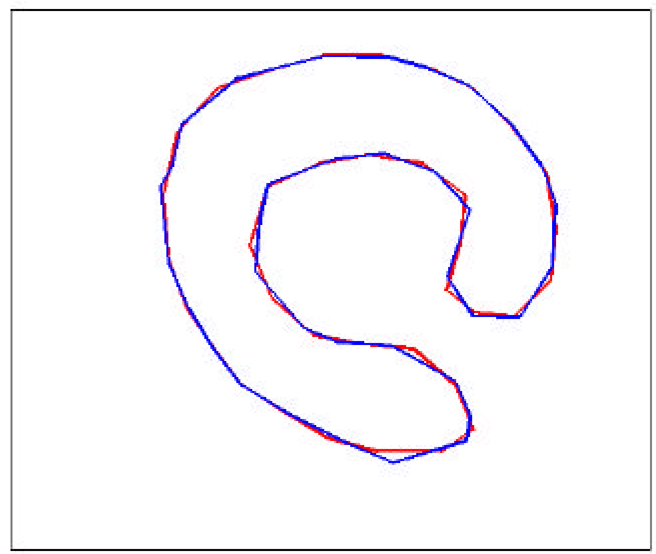
t_0 t_1 t_2 t_3 t_4 t_5
object
projection data from actual object
projection data from reconstructed object
Actual object (red)
Reconstructed object (blue)
100 point object
50 sample data sets
6 time views
0 parameters in F



t_0 t_1 t_2 t_3 t_4 t_5
object
projection data
from actual object
projection data from
reconstructed object
Actual object (red)
Reconstructed object (blue)
50 point object
50 sample data sets
6 time views
0 parameters in F



Actual object (red)
Reconstructed object (blue)

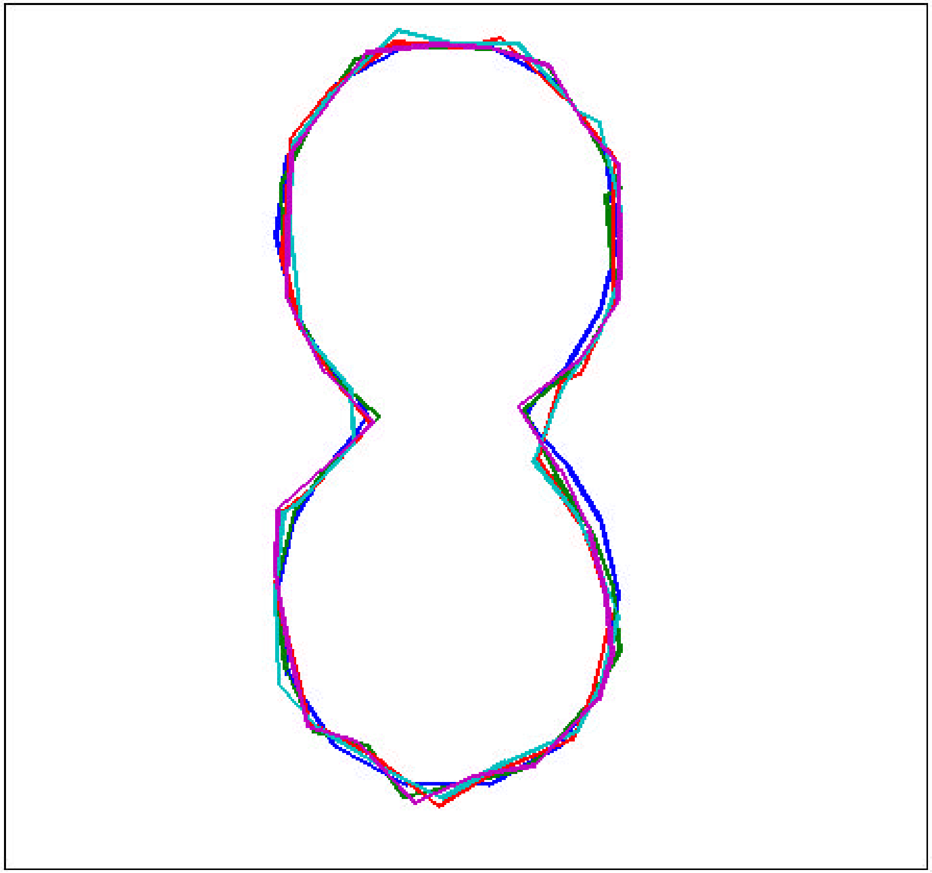


```
36 point object
50 sample data sets
6  time views
0  parameters in F
```

Reconstructions From Noisy Data

Simulated Random Particle-Counting Noise
(e.g. radiographic data)

'Thick' projections have larger noise than 'thin' projections.



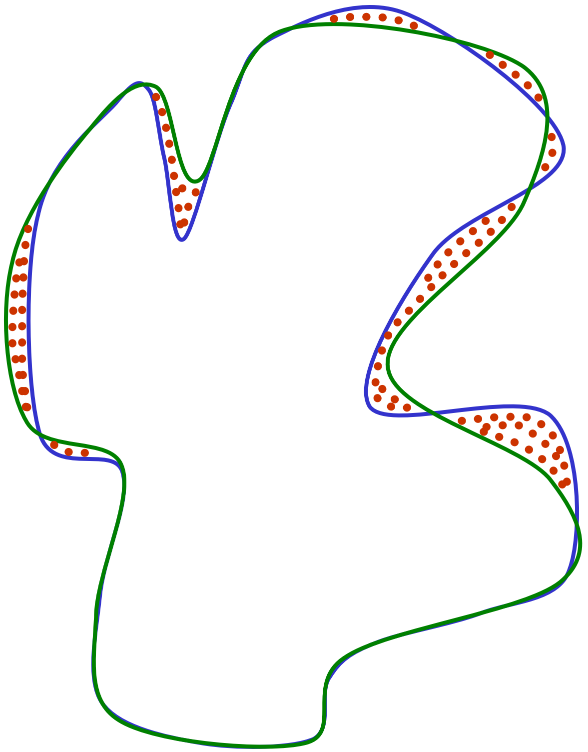
Object = blue
1%-5% = green
1%-10% = red
1%-15% = cyan
1%-20% = magenta

Creating a Object-Reconstruction Merit Function

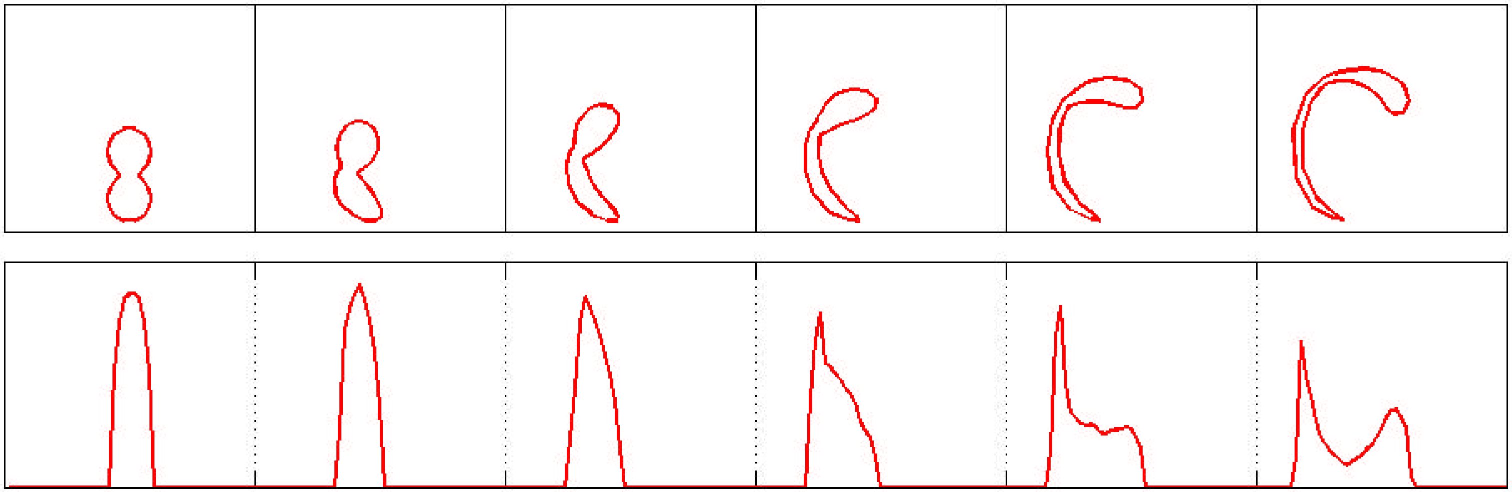
Consider an initial object (green) and the best found reconstruction (blue) from dynamics and projection data.

Let the merit function be:

$$g = \frac{difference area\,(red)}{object\ area}$$



Reconstruction Quality vs. Number of Views



object points = 30
data samples per projection = 20
restart iterations = 30

$$g = \frac{difference\,area}{object\ area}$$

$$f = \sum_k \left\| d_k - PF^k z_0 \right\|$$

V	g	*f*
2	0.372	0.0001
3	0.377	0.0045
4	0.315	0.0078
5	0.245	0.0045
6	0.202	0.0067
7	0.161	0.0050

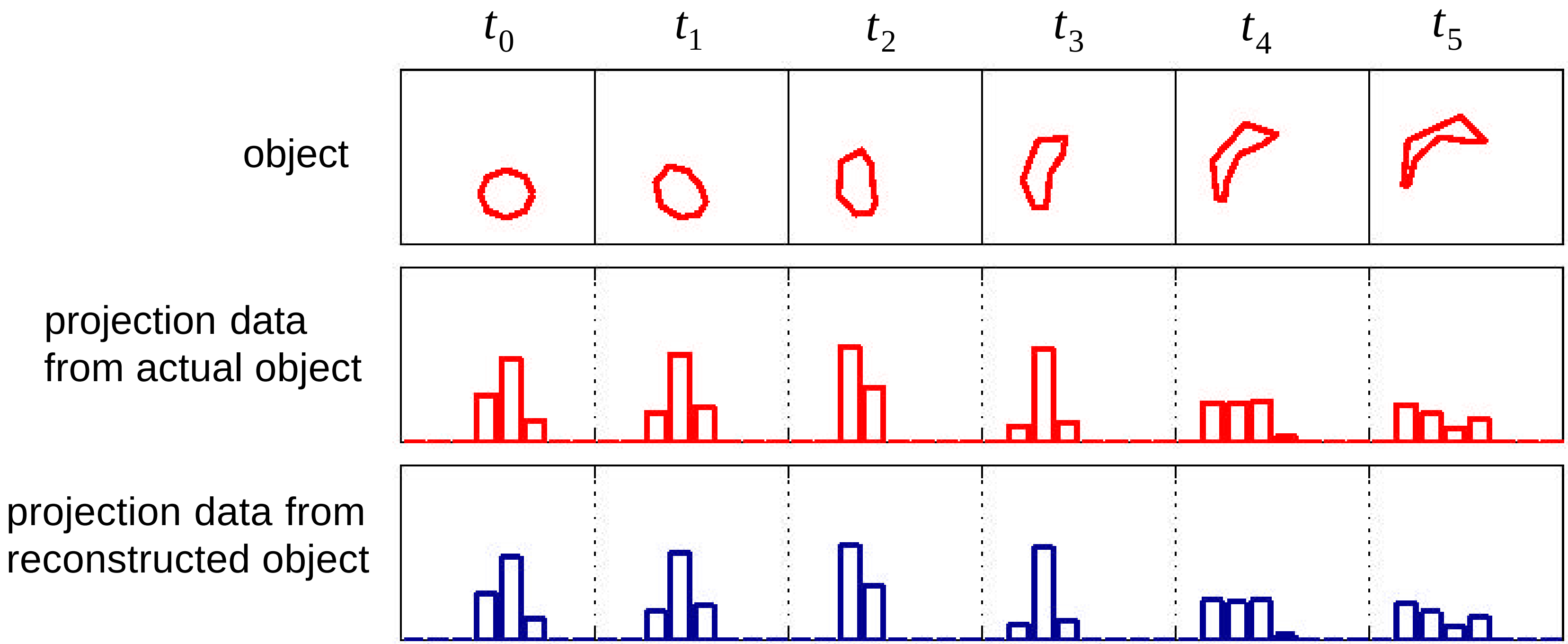
Estimating Parameters of the Dynamics

Can information about the underlying physics be obtained simultaneously with the object description?

Consider a time evolution operator $F = F(\lambda, t)$ that is unknown modulo a set of parameters λ. We might modify our advection field …

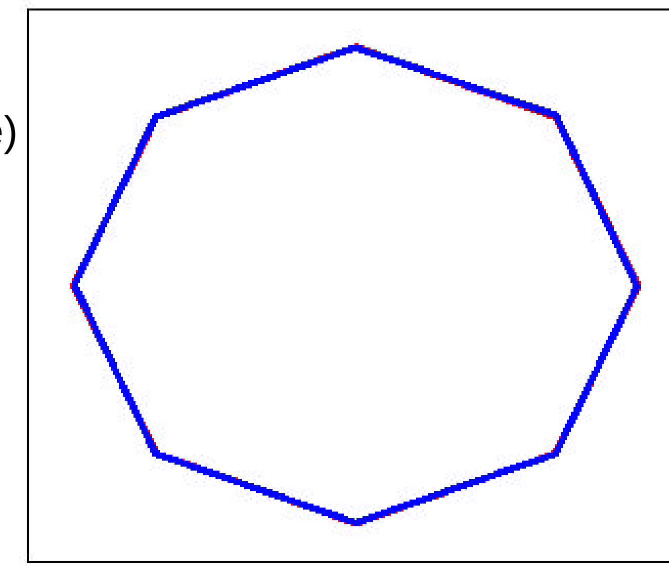
$$\dot{x} = -l_1 \sin^2(x)\sin(2y) + l_3 + l_5 x + l_7 y$$

$$\dot{y} = +l_2 \sin^2(y)\sin(2x) + l_4 + l_6 x + l_8 y$$

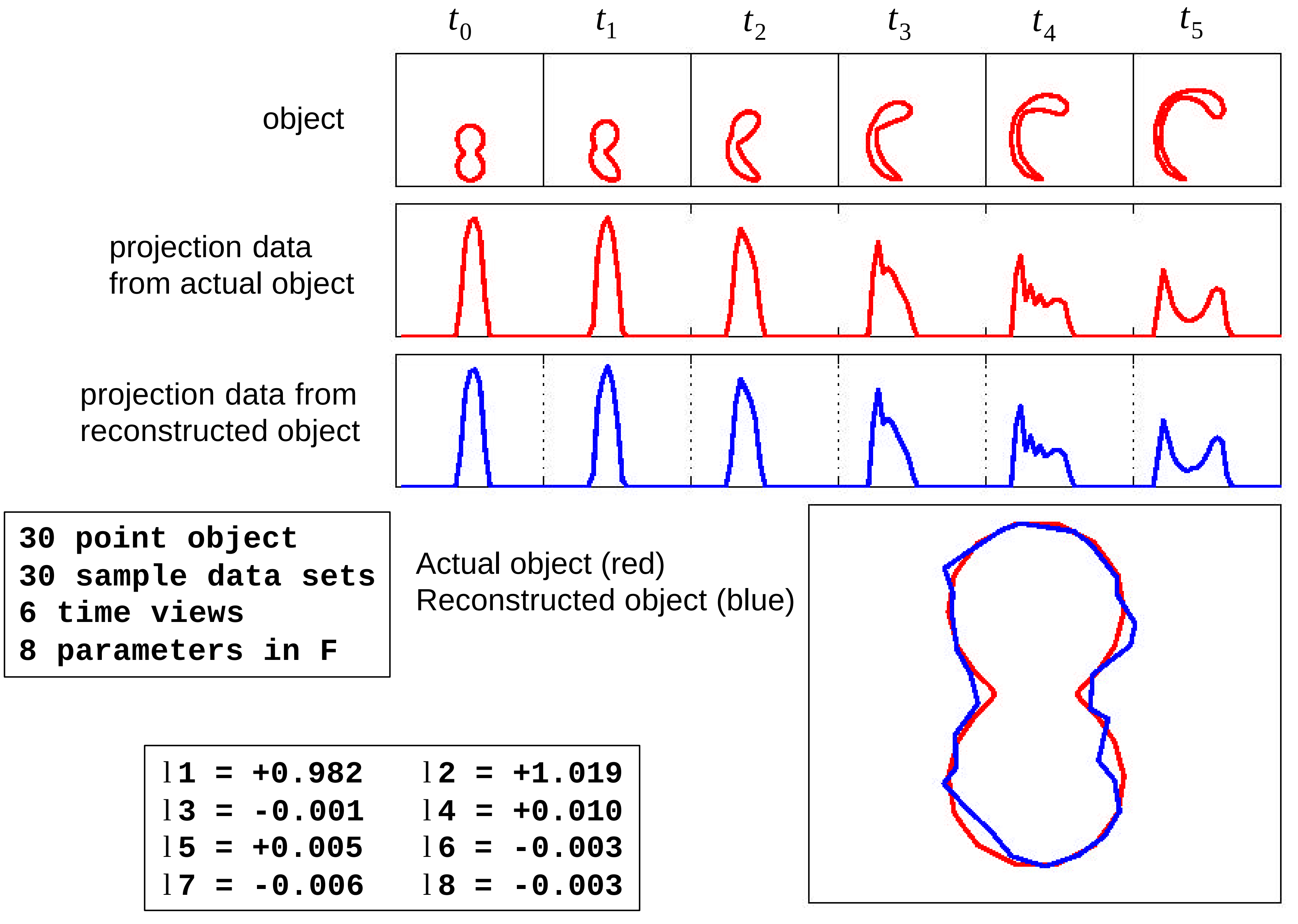



```
8 point object
8 sample data sets
6 time views
8 parameters in F
```

Actual object (red)
Reconstructed object (blue)



```
l1 = +1.000    l2 = +0.999
l3 = +0.000    l4 = +0.000
l5 = -0.000    l6 = +0.000
l7 = -0.000    l8 = -0.000
```



t_0 t_1 t_2 t_3 t_4 t_5
object
projection data from actual object
projection data from reconstructed object
30 point object
30 sample data sets
6 time views
8 parameters in F
Actual object (red)
Reconstructed object (blue)
l1 = +0.982 l2 = +1.019
l3 = -0.001 l4 = +0.010
l5 = +0.005 l6 = -0.003
l7 = -0.006 l8 = -0.003

Some Final Thoughts and Directions

Thus far the objects and dynamics are simply defined.
But the success in recovering them has been very good.

Other object parameterizations have not been explored.

This approach cannot easily handle topological changes in objects.
One possible path is to explore level-set methods.

Things of some importance not yet done …

- application to real data
- study of error propagation
- coding optimization
- application to hydrocodes